Convergent and divergent series, solutions of the Prolate Spheroidal differential equation

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Abstract

The prolate spheroidal wave functions, $\{\varphi_{n,\sigma,\tau}\}$, constitute an orthonormal basis of the space of σ-bandlimited functions on the real line, i. e. functions whose Fourier transforms have support on the interval $[-\sigma,\sigma]$. They can be characterized as the eigenfunctions of a differential operator of order 2:

$$(\tau^2-t^2)\varphi''_{n,\sigma,\tau}-2t\varphi'_{n,\sigma,\tau}-\sigma^2t^2\varphi_{n,\sigma,\tau}=\mu_{n,\sigma,\tau}\varphi_{n,\sigma,\tau}.$$

In this talk we will present some new results obtained on the formal solutions of this equation. For this purpose, we specialize to particular values of the two parameters: $\sigma=\tau=1$. We use the MAPLE package DESIR to compute the formal solutions in the neighborhood of the singularities (the regular ones ±1, and the irregular one, infinity) and to do some numerical experiments: computation of Stokes matrices [1] and of monodromy. This leads to the conjecture that the following properties are equivalent:

- μ is an eigenvalue of the differential operator $L=(t^2-1)\dfrac{d^2}{dt^2}+2t\dfrac{d}{dt}+t^2$;
- the series solutions near ±1 of the equation $L(y)=\mu y$ are entire functions (and so, eigenfunctions);
- the series appearing in the solutions near infinity of the equation $L(y)=\mu y$ are convergent;
- the Stokes phenomenon of the operator $L-\mu$ at infinity is trivial;
- the monodromy around $[-1,1]$ of the operator $L-\mu$ is trivial.

The second part of the talk will give the proof of the conjecture.

References

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