Symbolic Methods in Studying Linear Hamiltonian Systems of Differential Equations with Periodic Coefficients

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We consider a linear hamiltonian system of differential equations

$$\frac{dx}{dt} = JH(t,\varepsilon)x, \tag{1}$$

where $x^T = (x_1, x_2, \ldots, x_{2n})$ is a $2n$-dimensional vector whose components x_k and x_{n+k} are the canonically conjugated coordinate and momentum, $J = \begin{pmatrix} 0 & E_n \\ -E_n & 0 \end{pmatrix}$ and E_n is the $n \times n$ identity matrix, $H(t,\varepsilon)$ is the real-valued $2n \times 2n$ matrix function which can be represented in the form of the converging series

$$H(t,\varepsilon) = H_0 + \varepsilon H_1(t) + \varepsilon^2 H_2(t) + \ldots, \tag{2}$$

where ε is a small parameter. The matrix functions $H_k(t)$ $(k = 1, 2, \ldots)$ in (2) are continuous and periodic with a period T, while H_0 is a constant matrix. Besides, H_0 and $H_k(t)$ can depend on one or two parameters. Equations of the form (1) describe dynamical systems with intrinsic periodicity and appear in many branches of science and engineering.

According to the general theory of differential equations with periodic coefficients, behaviour of solutions of the system (1) is determined by its characteristic exponents. Therefore, studying such a system usually starts from calculation of the characteristic exponents which can be either purely imaginary numbers or complex numbers with nonzero real part.

The main purpose of this talk is to demonstrate different algorithms for symbolic calculation of characteristic exponents for the system (1) and determination of the domains in the parameter space, where characteristic exponents are purely imaginary numbers. It is planned also to present an algorithm for constructing periodic and quasi-periodic solutions of the system (1) in the form of Fourier polynomials. All the algorithms are implemented with the computer algebra system *Mathematica* and elliptic restricted many-body problems are used as examples.